後 卓 越 計 畫 進 度 報 告

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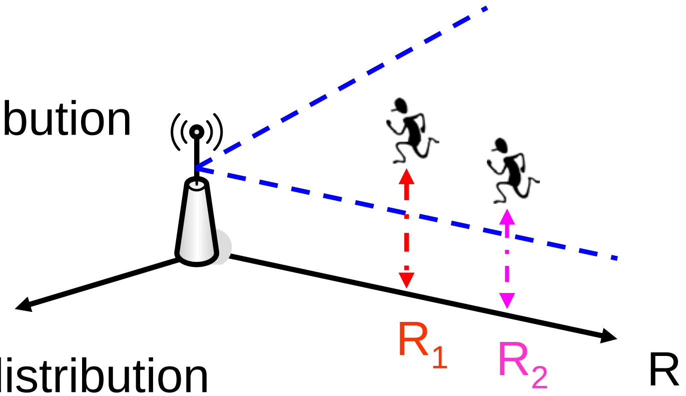
Why Tracking and Prediction Are Needed in an Indoor Location Estimation (1/2)

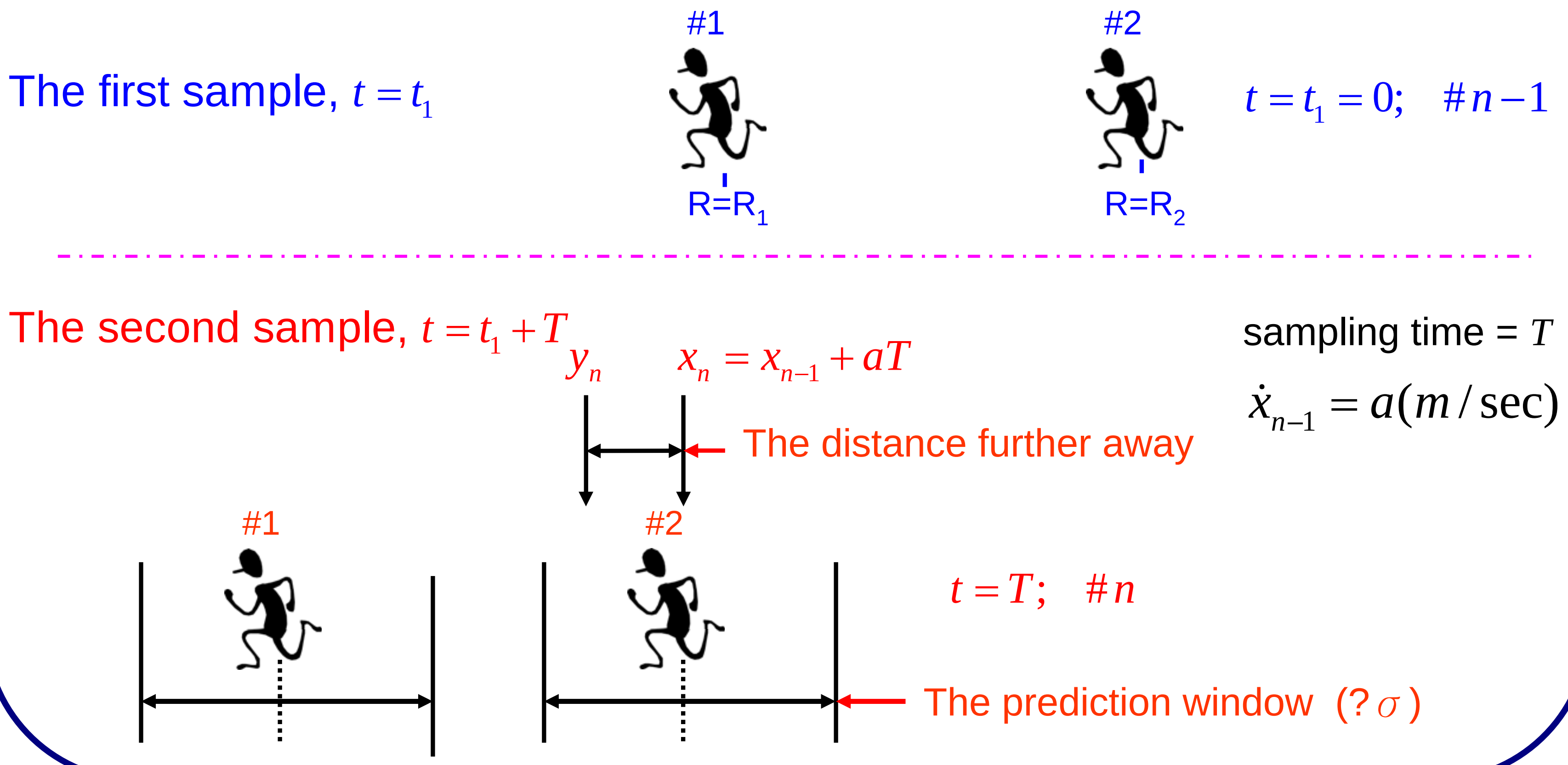
- Representation of Dynamic Systems
  - ✓ State equation: $\mathbf{x}_k = f_x(\mathbf{x}_{k-1}, \mathbf{u}_k)$
    - $\mathbf{x}_k$ state vector at time instant k
    - f_x state transition function
    - $\mathbf{u}_k$ process noise with known distribution
  - ✓ Observation equation: $\mathbf{z}_k = f_z(\mathbf{x}_k, \mathbf{v}_k)$
    - $\mathbf{z}_k$ observations at time instant k
    - f_z observation function
    - $\mathbf{v}_k$ observation noise with known distribution



- Assume that on the first sample at time $t=t_1$, two people are detected at ranges R_1 and R_2; assume that on the next sample at time $t=t_1+T$, again two people are detected
  - – The question arises as to whether these two people detected on the second sample are the same two people or two new people

Why Tracking and Prediction Are Needed in an Indoor Location Estimation (2/2)

- On the n-th sample, the predicted and measured positions are x_n and y_n, respectively



Kalman Filter (KF) Model

Target motion

$$\begin{bmatrix} x_1(t+T) \\ x_2(t+T) \\ \dot{x}_1(t+T) \\ \dot{x}_2(t+T) \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ \dot{w}_1(t) \\ \dot{w}_2(t) \end{bmatrix}$$

Position measurement

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

State equation

$$\mathbf{x}(t+T) = \mathbf{\Phi}(T)\mathbf{x}(t) + \mathbf{w}(t) \ , \quad \mathbf{Q}(t) = E\left[\mathbf{w}(t)\mathbf{w}(t)^T\right]$$

Observation equation

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{v}(T) \ , \quad \mathbf{R}(t) = E\left[\mathbf{v}(t)\mathbf{v}(t)^T\right]$$

State and measurement equations

$$\mathbf{z}_k = \mathbf{y}_k - \mathbf{H}_k \tilde{\mathbf{x}}_k$$ innovation = measurement – prediciton

$$\hat{\mathbf{x}}_k = \tilde{\mathbf{x}}_k + \mathbf{K}_k \mathbf{z}_k$$ State estimate

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{\Phi}_k \hat{\mathbf{x}}_k + \mathbf{f}_k$$ State prediction

Kalman gain and error covariance equations

$$\mathbf{K}_k = \tilde{\mathbf{P}}_k \mathbf{H}_k^T \left[\mathbf{H}_k \tilde{\mathbf{P}}_k \mathbf{H}_k^T + \mathbf{R}_k\right]^{-1}$$ Kalman gain

$$\hat{\mathbf{P}}_k = \left[1 - \mathbf{K}_k \mathbf{H}_k\right] \tilde{\mathbf{P}}_k$$ Estimate covariance

$$\tilde{\mathbf{P}}_{k+1} = \mathbf{\Phi}_k \hat{\mathbf{P}}_k \mathbf{\Phi}_k^T + \mathbf{Q}_k$$ Prediction covariance

α - β Filter Model

$\tilde{\mathbf{x}}, \tilde{\mathbf{v}} = \dot{\tilde{\mathbf{x}}}$ predicted position, velocity

$\hat{\mathbf{x}}, \hat{\mathbf{v}} = \dot{\hat{\mathbf{x}}}$ updated position, velocity

$\mathbf{y}_k$ measured position

α and β are tuning constants between 0 and 1

Prediction step

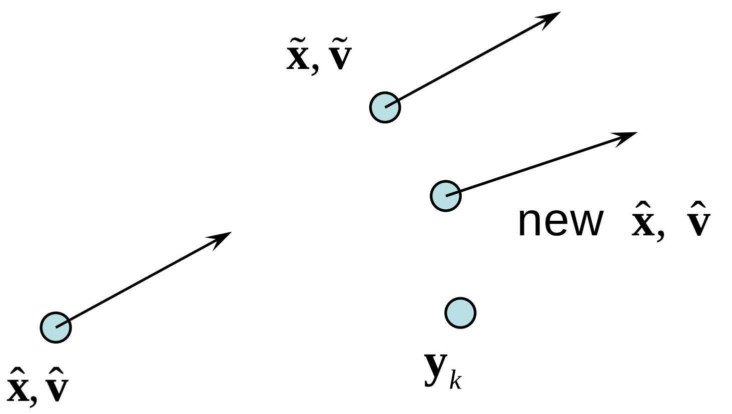
$$\tilde{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + T\hat{\mathbf{v}}_k$$

$$\tilde{\mathbf{v}}_k = \hat{\mathbf{v}}_k$$

Updating step

$$\hat{\mathbf{x}} = \tilde{\mathbf{x}}_k + \alpha\left(\mathbf{y}_k - \tilde{\mathbf{x}}_k\right)$$

$$\hat{\mathbf{v}}_k = \hat{\mathbf{v}}_{k-1} + \beta\frac{(\mathbf{y}_k - \tilde{\mathbf{x}}_k)}{T}$$



$\alpha = \beta = 0$: Measurement has no effect

$\alpha = \beta = 1$: History has no effect

Properties

- Properties of the α-β tracker :
  - The α-β tracker is a low-pass filter used to smooth out the effects of measurement noise
  - If the α-β tracker is based on the steady state of the KF algorithm, it does not require a priori knowledge of the state and measurement noise covariances
  - The performance of the α-β filter may not be as good as that of the KF
  - The equations of the α-β filter are simpler to implement than that of the KF
- The degenerate form of the KF is taken as follows:

Measurement matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Coefficient (gain) matrix

$$\mathbf{K} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \\ \beta/T & 0 \\ 0 & \beta/T \end{bmatrix}$$